CUSUM Chart

The cumulative sum chart, unlike the previously discussed SPC charts (Shewart charts) reflects the results of all of the samples rather than single sample values. It plots the cumulative sum of deviations from the mean or nominal specified value. If a process is going out of control, the sum will progressively go more positive or negative across the samples. If there are M samples, the cumulative sum S is given as

$$S = \sum_{i=1}^{M} (\bar{X}_i - \mu_o)$$

Where $\bar{X}_i$ is the observed sample mean and μ_o is the nominal value or (overall mean.)

It is often desirable to draw some boundaries to indicate when a process is out of control. By convention we use a standardized difference to specify this value. For example with the boltsize.LAZ data, we might specify that we wish to be sensitive to a difference of 0.02 from the mean. To standardize this value we obtain

$$\delta = \frac{0.02}{\sigma_x}$$

or using our sample values as estimates obtain

$$\delta = \frac{0.02}{S_x} = \frac{0.02}{0.359} = 0.0557$$

A "V Mask" is then drawn starting at a distance "d" from the last cumulative sum value with an angle θ back toward the first sample deviation. In order to calculate the distance d we need to know the probabilities of a Type I and Type II error, that is, the probability □ of incorrectly concluding that a shift to out-of-control has taken place and the probability □ of failing to detect an out-of-control condition. If these values are specified then we can obtain the distance d as

$$d = \left(\frac{2}{\delta^2}\right) \ln \left(\frac{1 - \beta}{\alpha}\right)$$

When you run the CUSUM procedure you will note that the alpha and beta error rates have been set to default values of 0.05 and 0.20. This would imply that an error of the first type (concluding out-of-control when in fact it is not) is a more "expense" error than concluding that the process is in control when in fact it is not. Depending on the cost of shut-down and correction of the process versus scraping of parts out of tolerance, you may wish to adjust these default values.

The angle of the V mask is obtained by

$$\theta = \tan^{-1}\left(\frac{\alpha}{2k}\right)$$

where k is a scaling factor typically obtained as $k = 2\,\sigma_x$

The specification form for the CUSUM chart is shown below for the data file labeled boltsize.txt. We have specified our desire to detect shifts of 0.02 in the process and are using the 0.05 and 0.20 probabilities for the two types of errors.

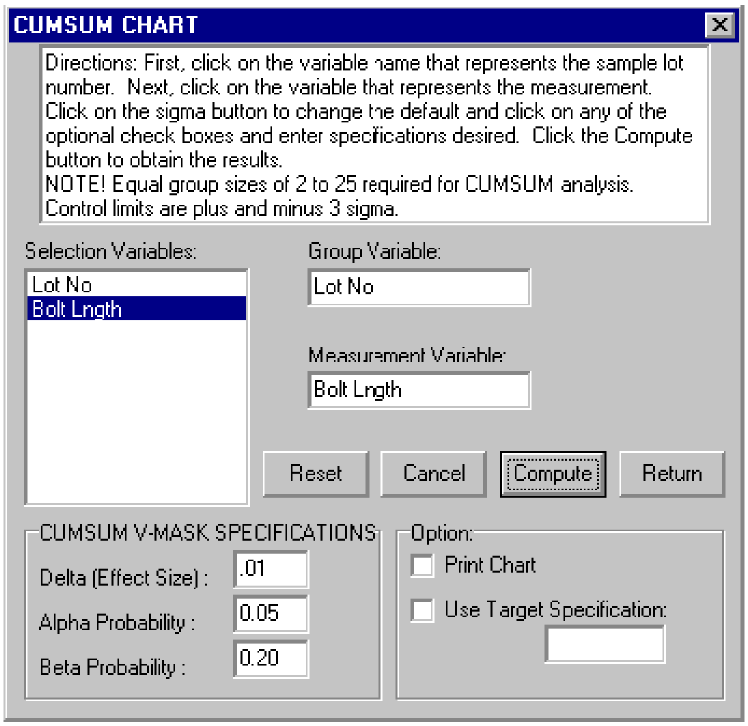

Figure 1 The SPC CUMSUM Chart Specification Dialog

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CUMSUM Chart Results

Group Size Mean      Std.Dev.  Cum.Dev. of
                               mean from Target

_____ ____ ________  ________  ___________
   1    5   19.88     0.37        -0.10
   2    5   19.90     0.29        -0.18
   3    5   20.16     0.27         0.00
   4    5   20.08     0.29         0.10
   5    5   19.88     0.49         0.00
   6    5   19.90     0.39        -0.08
   7    5   20.02     0.47        -0.04
   8    5   19.98     0.43        -0.04
Mean of group deviations = -0.005
Mean of all observations = 19.975
Std. Dev. of Observations =    0.359
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Standard Error of Mean =    0.057
Target Specification = 19.980
Lower Control Limit =   19.805, Upper Control Limit =   20.145
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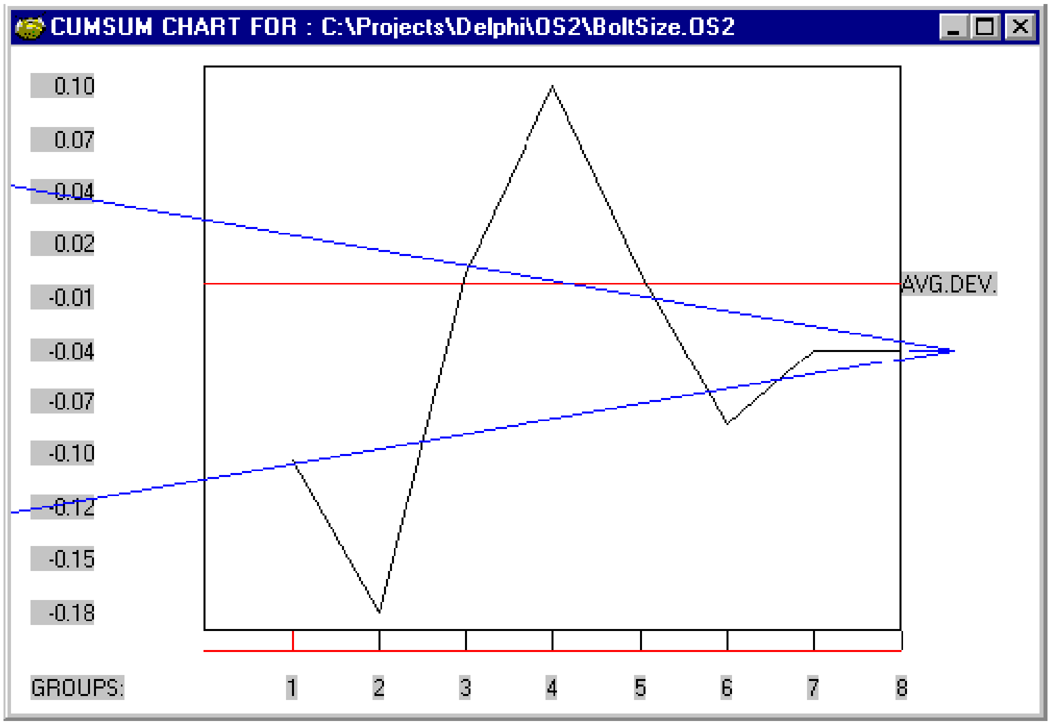

Figure 2   An SPC CUMSUM Chart