

## Kolmogorov-Smirnov Test

One often is interested in comparing a distribution of observed values with a theoretical distribution of values. Because many statistical tests assume a “normal” distribution, a variety of tests have been developed to determine whether or not two distributions are different beyond that expected due to random sampling variations. This test lets you compare your distribution with several theoretical distributions. We have loaded the file labeled KSTest.LAZ as an example. Below is the dialog form used to specify our analysis. In this example the file contains values, frequency of values and a group code for comparing the frequency distribution of the two groups.

**Kolmogorov-Smirnov Test**

Available Variables:

Comparison to:

- ☒ Observed variable distribution
- ☐ equal (rectangular) distribution
- ☐ Normal (Gaussian) distribution
- ☐ Student t distribution
- ☐ Chi-squared distribution
- ☐ Poisson distribution

Values variable: Value

Group Code (1 or 2) Group

Frequency variable 1: Freq

Input Type:

- ☐ Values to be counted
- ☒ Values and their frequencies

This procedure is used to test the difference between an observed score distribution and either another distribution or a theoretical distribution. The user may elect to enter values to be counted or values and their corresponding frequency. A variable containing frequencies of values should be defined as integers. When comparing two distributions, you will create the following variables: (1) a group code variable with values of 1 or 2 to indicate the two distributions, (2) a values variable containing the value of the distribution, and (3) a variable containing the frequency of a value for the first or second group.

Steps to the analysis:

1. Select the input type option of Values to be Counted or Read Values and their Frequencies.
2. Select the comparison to option (another observed variable or a theoretical variable).
3. Select the variable (or variables) to be analyzed.
4. Select options for printing and / or plotting distribution characteristics
5. Click the Compute button.

Options:

- ☒ Plot the observed distribution and the comparison distribution
- ☒ Print Observed and Comparison Probabilities

Reset Cancel Compute Return

**Figure 1. Kolmogorov-Smirnov Test of Similar Distributions**

One is then presented with a dialog form for specifying the interval size in which to group the observed scores. The number of resulting categories must be equal to or less than the number of cases observed.

**Freq. Dist. Specifications**

Minimum

Maximum

Range

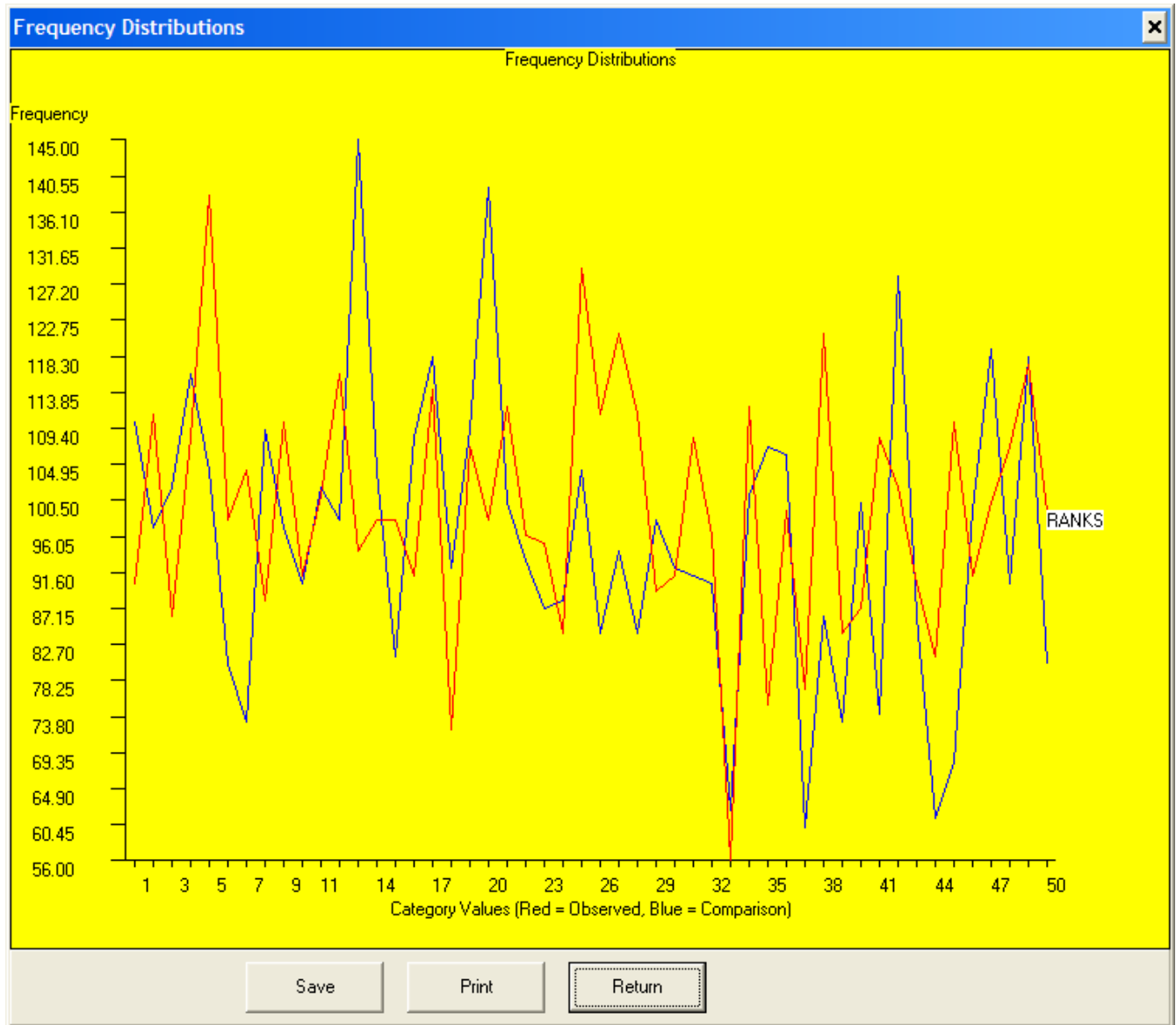
The interval size and number of intervals are shown below. You may change the number of intervals by entering a new interval size. Click on the current interval size and enter a new value. Press return when finished.

Interval Size:       Number of Intervals:

**Figure 1. Specification for Interval Size**

The results for our sample are shown below:



**Figure 2. Plot of Distributions in the Kolmogorov-Smirnov Test**

Kolmogorov-Smirnov Test

Analysis of variable Value

FROM	UP TO	FREQ.	PCNT	CUM.FREQ.	CUM.PCNT.	%ILE	RANK
1.00	2.00	90.000	0.018	90.000	0.018	0.009	
2.00	3.00	111.000	0.022	201.000	0.040	0.029	
3.00	4.00	86.000	0.017	287.000	0.058	0.049	
4.00	5.00	108.000	0.022	395.000	0.079	0.069	
5.00	6.00	138.000	0.028	533.000	0.107	0.093	
6.00	7.00	98.000	0.020	631.000	0.127	0.117	
7.00	8.00	104.000	0.021	735.000	0.148	0.137	
8.00	9.00	88.000	0.018	823.000	0.166	0.157	
9.00	10.00	110.000	0.022	933.000	0.188	0.177	
10.00	11.00	91.000	0.018	1024.000	0.206	0.197	
11.00	12.00	101.000	0.020	1125.000	0.226	0.216	
12.00	13.00	116.000	0.023	1241.000	0.250	0.238	

13.00	14.00	94.000	0.019	1335.000	0.269	0.259
14.00	15.00	98.000	0.020	1433.000	0.288	0.278
15.00	16.00	98.000	0.020	1531.000	0.308	0.298
16.00	17.00	91.000	0.018	1622.000	0.326	0.317
17.00	18.00	114.000	0.023	1736.000	0.349	0.338
18.00	19.00	72.000	0.014	1808.000	0.364	0.356
19.00	20.00	107.000	0.022	1915.000	0.385	0.374
20.00	21.00	98.000	0.020	2013.000	0.405	0.395
21.00	22.00	112.000	0.023	2125.000	0.427	0.416
22.00	23.00	96.000	0.019	2221.000	0.447	0.437
23.00	24.00	95.000	0.019	2316.000	0.466	0.456
24.00	25.00	84.000	0.017	2400.000	0.483	0.474
25.00	26.00	129.000	0.026	2529.000	0.509	0.496
26.00	27.00	111.000	0.022	2640.000	0.531	0.520
27.00	28.00	121.000	0.024	2761.000	0.555	0.543
28.00	29.00	111.000	0.022	2872.000	0.578	0.566
29.00	30.00	89.000	0.018	2961.000	0.596	0.587
30.00	31.00	91.000	0.018	3052.000	0.614	0.605
31.00	32.00	108.000	0.022	3160.000	0.636	0.625
32.00	33.00	96.000	0.019	3256.000	0.655	0.645
33.00	34.00	56.000	0.011	3312.000	0.666	0.660
34.00	35.00	112.000	0.023	3424.000	0.689	0.677
35.00	36.00	75.000	0.015	3499.000	0.704	0.696
36.00	37.00	99.000	0.020	3598.000	0.724	0.714
37.00	38.00	77.000	0.015	3675.000	0.739	0.731
38.00	39.00	121.000	0.024	3796.000	0.763	0.751
39.00	40.00	84.000	0.017	3880.000	0.780	0.772
40.00	41.00	87.000	0.017	3967.000	0.798	0.789
41.00	42.00	108.000	0.022	4075.000	0.820	0.809
42.00	43.00	102.000	0.021	4177.000	0.840	0.830
43.00	44.00	90.000	0.018	4267.000	0.858	0.849
44.00	45.00	81.000	0.016	4348.000	0.874	0.866
45.00	46.00	110.000	0.022	4458.000	0.897	0.886
46.00	47.00	91.000	0.018	4549.000	0.915	0.906
47.00	48.00	100.000	0.020	4649.000	0.935	0.925
48.00	49.00	107.000	0.022	4756.000	0.957	0.946
49.00	50.00	117.000	0.024	4873.000	0.980	0.968
50.00	51.00	99.000	0.020	4972.000	1.000	0.990

# Kolmogorov-Smirnov Test

## Analysis of Comparison Group

FROM	UP TO	FREQ.	PCNT	CUM.FREQ.	CUM.PCNT.	%ILE RANK
1.00	2.00	110.000	0.023	110.000	0.023	0.011
2.00	3.00	97.000	0.020	207.000	0.043	0.033
3.00	4.00	102.000	0.021	309.000	0.064	0.054
4.00	5.00	116.000	0.024	425.000	0.089	0.077
5.00	6.00	104.000	0.022	529.000	0.110	0.099
6.00	7.00	80.000	0.017	609.000	0.127	0.119
7.00	8.00	73.000	0.015	682.000	0.142	0.135
8.00	9.00	109.000	0.023	791.000	0.165	0.154
9.00	10.00	97.000	0.020	888.000	0.185	0.175
10.00	11.00	90.000	0.019	978.000	0.204	0.194
11.00	12.00	102.000	0.021	1080.000	0.225	0.215
12.00	13.00	98.000	0.020	1178.000	0.246	0.235
13.00	14.00	145.000	0.030	1323.000	0.276	0.261
14.00	15.00	104.000	0.022	1427.000	0.297	0.287

15.00	16.00	81.000	0.017	1508.000	0.314	0.306
16.00	17.00	108.000	0.023	1616.000	0.337	0.326
17.00	18.00	118.000	0.025	1734.000	0.361	0.349
18.00	19.00	92.000	0.019	1826.000	0.381	0.371
19.00	20.00	109.000	0.023	1935.000	0.403	0.392
20.00	21.00	139.000	0.029	2074.000	0.432	0.418
21.00	22.00	100.000	0.021	2174.000	0.453	0.443
22.00	23.00	93.000	0.019	2267.000	0.473	0.463
23.00	24.00	87.000	0.018	2354.000	0.491	0.482
24.00	25.00	88.000	0.018	2442.000	0.509	0.500
25.00	26.00	104.000	0.022	2546.000	0.531	0.520
26.00	27.00	84.000	0.018	2630.000	0.548	0.540
27.00	28.00	94.000	0.020	2724.000	0.568	0.558
28.00	29.00	84.000	0.018	2808.000	0.585	0.577
29.00	30.00	98.000	0.020	2906.000	0.606	0.596
30.00	31.00	92.000	0.019	2998.000	0.625	0.615
31.00	32.00	91.000	0.019	3089.000	0.644	0.634
32.00	33.00	90.000	0.019	3179.000	0.663	0.653
33.00	34.00	62.000	0.013	3241.000	0.676	0.669
34.00	35.00	101.000	0.021	3342.000	0.697	0.686
35.00	36.00	107.000	0.022	3449.000	0.719	0.708
36.00	37.00	106.000	0.022	3555.000	0.741	0.730
37.00	38.00	60.000	0.013	3615.000	0.754	0.747
38.00	39.00	86.000	0.018	3701.000	0.772	0.763
39.00	40.00	73.000	0.015	3774.000	0.787	0.779
40.00	41.00	100.000	0.021	3874.000	0.808	0.797
41.00	42.00	74.000	0.015	3948.000	0.823	0.815
42.00	43.00	128.000	0.027	4076.000	0.850	0.836
43.00	44.00	86.000	0.018	4162.000	0.868	0.859
44.00	45.00	61.000	0.013	4223.000	0.880	0.874
45.00	46.00	68.000	0.014	4291.000	0.895	0.887
46.00	47.00	99.000	0.021	4390.000	0.915	0.905
47.00	48.00	119.000	0.025	4509.000	0.940	0.928
48.00	49.00	90.000	0.019	4599.000	0.959	0.949
49.00	50.00	118.000	0.025	4717.000	0.983	0.971
50.00	51.00	80.000	0.017	4797.000	1.000	0.992

# Kolmogorov-Smirnov Analysis of Value and Comparison

Observed Mean = 25.042 for 9769 cases in 50 categories  
Standard Deviation = 14.491

# Kolmogorov-Smirnov Distribution Comparison

CATEGORY	OBSERVED	COMPARISON	DIFFERENCE
VALUE	CUM. PROB.	CUM. PROB.	CUM. PROB.
1	0.018	0.023	-0.005
2	0.040	0.043	-0.003
3	0.058	0.064	-0.007
4	0.079	0.089	-0.009
5	0.107	0.110	-0.003
6	0.127	0.127	-0.000
7	0.148	0.142	0.006
8	0.166	0.165	0.001
9	0.188	0.185	0.003
10	0.206	0.204	0.002
11	0.226	0.225	0.001
12	0.250	0.246	0.004

13	0.269	0.276	-0.007
14	0.288	0.297	-0.009
15	0.308	0.314	-0.006
16	0.326	0.337	-0.011
17	0.349	0.361	-0.012
18	0.364	0.381	-0.017
19	0.385	0.403	-0.018
20	0.405	0.432	-0.027
21	0.427	0.453	-0.026
22	0.447	0.473	-0.026
23	0.466	0.491	-0.025
24	0.483	0.509	-0.026
25	0.509	0.531	-0.022
26	0.531	0.548	-0.017
27	0.555	0.568	-0.013
28	0.578	0.585	-0.008
29	0.596	0.606	-0.010
30	0.614	0.625	-0.011
31	0.636	0.644	-0.008
32	0.655	0.663	-0.008
33	0.666	0.676	-0.010
34	0.689	0.697	-0.008
35	0.704	0.719	-0.015
36	0.724	0.741	-0.017
37	0.739	0.754	-0.014
38	0.763	0.772	-0.008
39	0.780	0.787	-0.006
40	0.798	0.808	-0.010
41	0.820	0.823	-0.003
42	0.840	0.850	-0.010
43	0.858	0.868	-0.009
44	0.874	0.880	-0.006
45	0.897	0.895	0.002
46	0.915	0.915	-0.000
47	0.935	0.940	-0.005
48	0.957	0.959	-0.002
49	0.980	0.983	-0.003
50	1.000	1.000	0.000
51	1.000	1.000	0.000

Kolmogorov-Smirnov Statistic D = 0.027 with probability > D = 0.000