

CUSUM Chart

The cumulative sum chart, unlike the previously discussed SPC charts (Shewart charts) reflects the results of all of the samples rather than single sample values. It plots the cumulative sum of deviations from the mean or nominal specified value. If a process is going out of control, the sum will progressively go more positive or negative across the samples. If there are M samples, the cumulative sum S is given as

$$S = \sum_{i=1}^M (\bar{X}_i - \mu_0) \quad \text{Where } \bar{X}_i \text{ is the observed sample mean and } \mu_0 \text{ is the nominal value or (overall mean.)}$$

It is often desirable to draw some boundaries to indicate when a process is out of control. By convention we use a standardized difference to specify this value. For example with the boltsize.LAZ data, we might specify that we wish to be sensitive to a difference of 0.02 from the mean. To standardize this value we obtain

$$\delta = \frac{0.02}{\sigma_x}$$

or using our sample values as estimates obtain

$$\delta = \frac{0.02}{S_x} = \frac{0.02}{0.359} = 0.0557$$

A “V Mask” is then drawn starting at a distance “d” from the last cumulative sum value with an angle θ back toward the first sample deviation. In order to calculate the distance d we need to know the probabilities of a Type I and Type II error, that is, the probability α of incorrectly concluding that a shift to out-of-control has taken place and the probability β of failing to detect an out-of-control condition. If these values are specified then we can obtain the distance d as

$$d = \left(\frac{2}{\delta^2} \right) \ln \left(\frac{1 - \beta}{\alpha} \right)$$

When you run the CUSUM procedure you will note that the alpha and beta error rates have been set to default values of 0.05 and 0.20. This would imply that an error of the first type (concluding out-of-control when in fact it is not) is a more “expense” error than concluding that the process is in control when in fact it is not. Depending on the cost of shut-down and correction of the process versus scraping of parts out of tolerance, you may wish to adjust these default values.

The angle of the V mask is obtained by

$$\theta = \tan^{-1}\left(\frac{\alpha}{2k}\right)$$

where k is a scaling factor typically obtained as $k = 2 \sigma_x$

The specification form for the CUSUM chart is shown below for the data file labeled boltsize.txt. We have specified our desire to detect shifts of 0.02 in the process and are using the 0.05 and 0.20 probabilities for the two types of errors.

CUSUM CHART

Directions: First, click on the variable name that represents the sample lot number. Next, click on the variable that represents the measurement. Click on the sigma button to change the default and click on any of the optional check boxes and enter specifications desired. Click the Compute button to obtain the results.
NOTE! Equal group sizes of 2 to 25 required for CUSUM analysis. Control limits are plus and minus 3 sigma.

Selection Variables:
 Lot No
Bolt Length

Group Variable:
 Lot No

Measurement Variable:
 Bolt Length

Reset Cancel **Compute** Return

CUSUM V-MASK SPECIFICATIONS

Delta (Effect Size) : .01
 Alpha Probability : 0.05
 Beta Probability : 0.20

Option:
☐ Print Chart
☐ Use Target Specification:

Figure 1 The SPC CUSUM Chart Specification Dialog

CUSUM Chart Results

Group	Size	Mean	Std.Dev.	Cum.Dev. of mean from Target
1	5	19.88	0.37	-0.10
2	5	19.90	0.29	-0.18
3	5	20.16	0.27	0.00
4	5	20.08	0.29	0.10
5	5	19.88	0.49	0.00
6	5	19.90	0.39	-0.08
7	5	20.02	0.47	-0.04
8	5	19.98	0.43	-0.04
Mean of group deviations = -0.005				
Mean of all observations = 19.975				
Std. Dev. of Observations = 0.359				

Standard Error of Mean = 0.057
Target Specification = 19.980
Lower Control Limit = 19.805, Upper Control Limit = 20.145

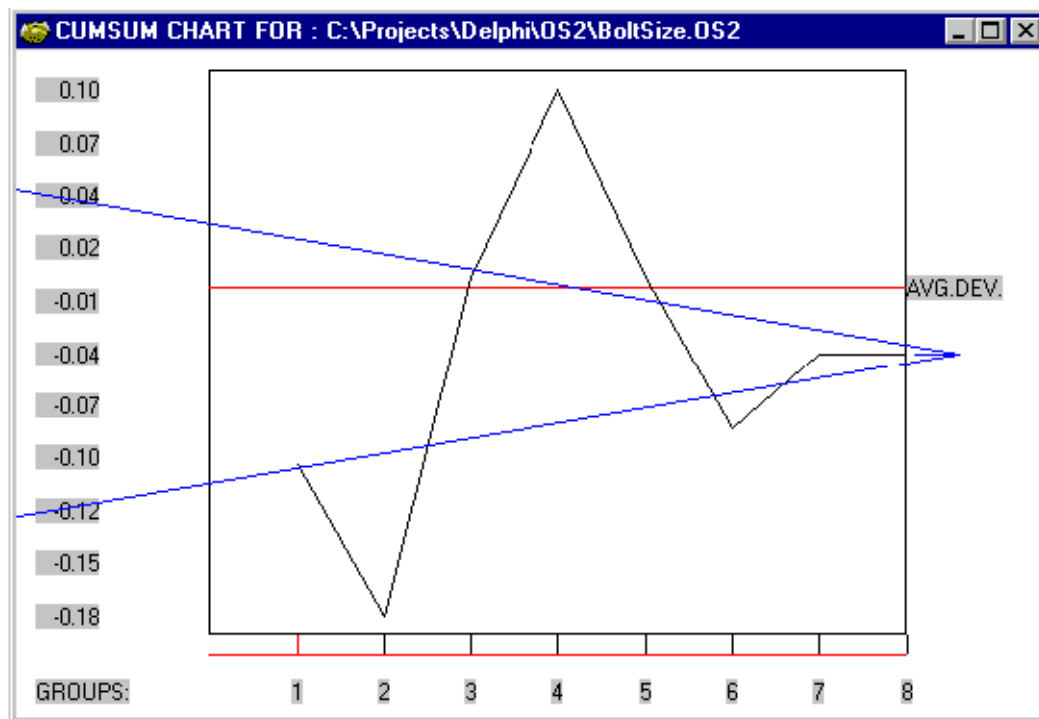


Figure 2 An SPC CUMSUM Chart